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ABSTRACT

Three examples of how the computer terminal can be used to provide the kind of "pictorial" information that makes graphs such a valuable aid in the teaching of quantitatively oriented subjects are presented. In the first example the graph is part of the "frame" of information presented to a student in a tutorial which reviews trigonometric functions and their graphical descriptions. The second example shows how professionally prepared "packages" can be useful, especially when the primary activity of the user is not concerned with computing as such. The third module, prepared by Project Solo, lists a sample program which includes the essential tools necessary to produce good graphs, but which itself produces a rather poor graph. The problems at the end of the module require putting these tools together with some ingenuity to produce a better graph. (JY)

PROJECT SOLO

AN EXPERIMENT IN REGIONAL COMPUTING
FOR SECONDARY SCHOOL SYSTEMS

University of Pittsburgh • Department of Computer Science • Pittsburgh, Pennsylvania 15213

Newsletter No. 5 October 26, 1970

Graph Modules

This issue of the newsletter will be accompanied by three examples of how the computer terminal can be used to provide the kind of "pictorial" information that makes graphs such a valuable aid in the teaching of quantitatively oriented subjects. The first example is the work of Frank Wimberly of Pitt, the second comes from Mrs. Cheryl Porter of Westinghouse High School, and the third is an example of a packaged routine accessible through the Com-Share system.

Graphing by Computer

The "hands on" use of computers for problem solving can require considerable skill in programming. When considering such use by students, we must ask whether they should be "handed" completely functioning programs to be used for solving the problems being studied, or whether they should be given only the basic tools (NBS statements) for discovering the needed program. Many students have said (for example in their comments about the NBS Primer) that they don't like imitating pre-written programs, and that they want the chance to explore a bit.

The enclosed module was written in an effort to help meet this preference. The approach taken is to list a sample program in the module which includes the essential tools necessary to produce good graphs, but which itself produces a rather poor graph. The problems at the end of the module require putting these tools together with some ingenuity, and the cover of the module--a graph of the sin and cos produced by an NBS program--provides an illustration of what can be done.

The module is based on work originally done by Reed Brown, a teacher from Ellwood City High School, who participated in the NSF Summer Workshop at Pitt.

Graphs as an Element in Computer Tutorials

The second example enclosed with this newsletter represents a quite different use of the graphing capability. In the first part of the program, the graph is part of the "frame" of information presented to a student in a tutorial which reviews trigonometric functions and their graphical descriptions. The program then invites the student to experiment with changing the parameters used to describe trigonometric functions (amplitude, frequency, and phase shift). He immediately sees the effect of such changes as evidenced in the graph produced by the computer. The computer thus gives the student an experimental laboratory tool for exploring abstract concepts. Mrs. Porter's idea will become even more valuable when manufactures provide us with more sophisticated display devices.

The Polynomial Fitting (POLFIT) Package

The third example (pages 6, 7, 8) illustrates how professionally prepared "packages" can be useful, especially when the primary activity of the user is not concerned with computing as such. For example, science students in a laboratory might not wish to spend time in writing graphing programs, yet they may wish to use the computer to gather and display data from their experiments, fitting appropriate curves to the data to see the trend of the experiment, or to extrapolate values.

* * * * * *

P.S. The "Graphing by Computer" Module is enclosed as a separate unit. We have a small extra quantity of these available if you can use them.

Excerpts from a student interaction with Mrs. Porter's tutorial.

LOAD 166CP /CIRCD/
RUN

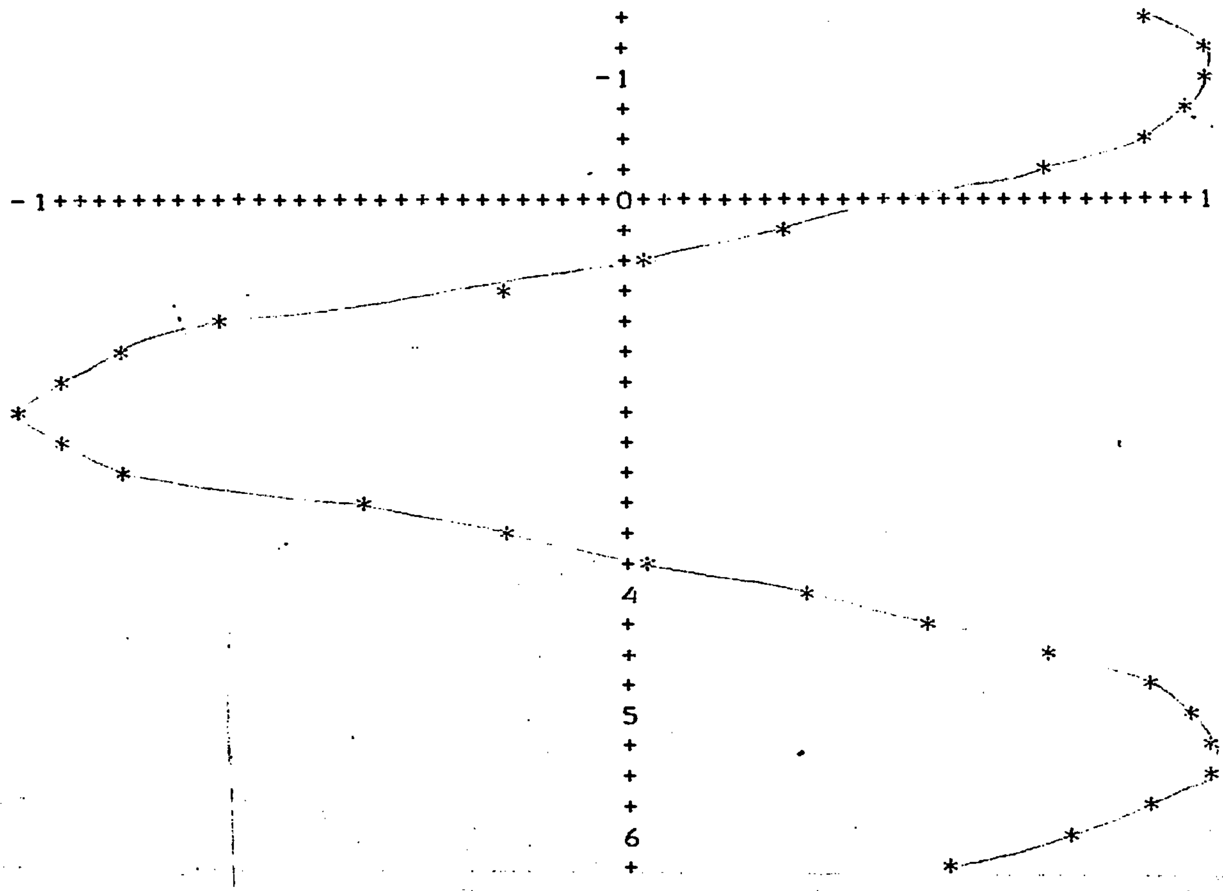
```
CIRCULAR FUNCTIØNS.  YØUR TASK IS TØ STUDY THE
GRAPH AND TØ TAKE NØTE ØF ITS RELATIØNSHIP TØ
THE CØØRDINATE AXES.  THEN YØU WILL BE ASKED
TØ WRITE THE EQUATIØN WHICH REPRESENTS THE PRØPER FUNCTIØN.
+                             + *
+                             +        *
-1                            +               *
+                             +                    *
+                             +                         *
+                             +                            *
0+++++++++++++++++++++++++++++1++++++++++++++++++++++++++++2
+                             +                            *
+                             +                         *
+                             +                    *
 1                            +               *
+                             +        *
.                             + *
+                       *     +
 2               *            +
+          *                  +
+    *                        +
+ *                           +
 3*                           +
+*                            +
+*                            +
+    *                        +
 4        *                   +
+               *             +
+                      *      +
+                             +*
 5                            +       *
+                             +              *
+                             +                    *
+                             +                        *
 6                            +                           *
+                             +                            *
LØØK AT THE GRAPH AND STUDY ITS PARTS.  REMEMBER
THAT THE Y-AXIS GØES ACRØSS THE PAPER AND THAT THE
X-AXIS GØES ALØNG THE PAPER AS IT MØVES THRØUGH THE
TERMINAL.  WHEN YØU THINK YØU KNØW WHAT FUNCTIØN
IS REPRESENTED, TYPE YØUR ANSWER BELØW.
?Y=1+SIN(X)
YØUR ANSWER IS NØT CØRRECT.  CHECK THE GRAPH TØ
DETERMINE ITS AMPLITUDE.  LØØK AT THE GRAPH IN
RELATIØN TØ THE X- AND Y-AXES.  THEN TRY TØ
ANSWER AGAIN.  TYPE IN YØUR ANSWER.
?Y=1+CØS(X)
GØØD--YØU ANSWERED CØRRECTLY.  THE FUNCTIØN CAN BE
EITHER Y=1+CØS(X) ØR Y=1+SIN(X-90).
```

CIRCD (continued)

```
NØW LET'S TRY ANØTHER GRAPH.  IF YØU ARE HAVING
TRØUBLE, REVIEW PAGE 2 ØF YØUR MØDULE.
*                              +
 *                             +
    *                         -1
          *                    +
                *              +
                       *       +
-3+++++++++++++++++++++++++++++0+++++++++++++++++++++++++++++3
                               +      *
                               +             *
                               +                   *
                               1                         *
                               +                            *
                               +                             *
                               +                             *
                               2                           *
                               +                       *
                               +                *
                               +          *
                               3   *
                           *   +
                    *          +
             *                 +
        *                      4
    *                          +
*                              +
*                              +
 *                             +
    *                          5
         *                     +
               *               +
                      *        6
                              *+
WHAT FUNCTIØN IS REPRESENTED BY THIS GRAPH?
?Y=3SIN(X)
Correct
HØW MANY CØMPLETE CYCLES ØF THIS CURVE ARE GRAPHED
BETWEEN 0 AND 360 DEGREES?
?2
NØ, LØØK AT THE CURVE.  THERE IS ØNLY ØNE CØMPLETE
PICTURE ØF THE SIN CURVE BETWEEN 0 DEGREES AND
360 DEGREES.  THEREFØRE, THE PERIØD IS UNCHANGED
AND IS EQUAL TØ 2 PI RADIANS.
THE FIRST TWØ GRAPHS WE LØØKED AT GAVE EXAMPLES ØF
CHANGES IN THE AMPLITUDE ØF THE SINE AND CØSINE
FUNCTIØNS AS A RESULT ØF THE ADDITIØN ØF A CØNSTANT
TØ THE EQUATIØN.  NØW LET'S LØØK AT WHAT EFFECT
CØNSTANTS HAVE ØN CHANGES IN THE PERIØD ØR SHIFTS IN
PHASE.  FRØM THE PRØBLEMS IN YØUR HANDØUT, TYPE IN THE
INFØRMATIØN AS REQUESTED.
DØ YØU WANT SIN ØR CØS?
?SIN
```

CIRCD (continued)

```
TYPE IN K3 FØR THE CHANGE IN PERIØD.
? 1
TYPE IN K4 FØR THE PHASE SHIFT.
? 45
```



(The student sees the graph of the function he has described)
etc.

The POLFIT package

On the next three pages we show a session at a terminal wherein a user acesses a packaged program that is available through one of the "dispatcher" systems provided by Com-Share. The one we show is called MATH, and it has 3 sub-programs on it called DIFF, POLFIT, and CFIT. As will be shown in our example, a user who is accessing one of these packages for the first time can get information on the commands available to him by saying HELP. After he becomes familiar with use of the routine, he will not ask to have such information printed, of course. Names of other dispatchers were included in the Com-Share technical newsletter sent to you recently. We have manuals describing the COGO (coordinate geometry civil engineering package), SUMS (statistical routines), and BUSINESS packages in the project office in case you would like to study them.

```
LAST LØGIN  NØV  2 16:08
-MATH      ← DISPATCHERS ARE CALLED FROM THE EXECUTIVE

PRØG: (LF) ← LINE FEED TO FIND OUT WHAT PROGRAMS ARE IN "MATH"
DIFF
PØLFIT
CFIT

PRØG: POLFIT ← DECIDES TO USE "PØLFIT"
VERSIØN DATE 7/5/70

←HELP ← ASKS FOR INFORMATION ON "PØLFIT"
7/5/70
LIST ØF VALID CØMMANDS

NEWS            GIVES A LIST ØF THE LATEST CHANGES TØ THE PRØGRAM

VERSIØN         TYPES ØUT THE LAST DATE ØF CHANGE TØ THE PRØGRAM

HELP            GIVES A LIST ØF VALID CØMMANDS AND A BRIEF EXPLANATIØN
                ØF THEIR FUNCTIØNS

ENTER           REQUESTS CØØRDINATES FRØM THE TTY

WRITE           WRITES THE CØØRDINATES ØUT TØ A DISC FILE

LØAD            LØADS CØØRDINATES FRØM A FILE CREATED WITH A WRITE
                CØMMAND

STATISTICS      TYPES ØUT THE MEAN VALUES ØF X AND Y AND THE STANDARD
                ERRØR ØF Y

LIST            LISTS THE CØØRDINATES THAT ARE CURRENTLY IN THE PRØGRAM

ADD             ALLØWS YØU TØ ADD CØØRDINATES

CHANGE          ALLØWS YØU TØ CHANGE CØØRDINATES

DELETE          ALLØWS YØU TØ DELETE CØØRDINATES

PØLFIT          FIT THE CØØRDINATES TØ A SPECIFIED DEGREE AND
                PRINTS THE INDEX ØF DETERMINATIØN

CØEFFICIENTS    TYPES ØUT THE CØEFFICIENTS ØF THE CØØRDINATES
                GIVEN

SUMMARY         PRINTS THE CØEFFICIENTS,X AND Y ACTUALLY CALCULATED
                AND THE DIFFERENCE BETWEEN Y ACTUAL, AND Y CALCULATED

ØUT             WRITES THE CØEFFICIENTS TØ A DISC FILE

END             STØPS PRØGRAM

EXIT            STØPS PRØGRAM

CHECK           CHECKS THE CØEFFICIENTS CURRENTLY IN THE PRØGRAM
                BY PLACING THEM IN AN EQUATIØN AND SØLVING THEM
                THRØUGH A RANGE YØU SPECIFY.  YØU HAVE THE ØPTIØN
                ØF EITHER PLØTTING THE RESULTS ØN A ØNE PAGE GRAPH
                ØR SEEING A LIST ØF THE CALCULATED PØINTS.
```

POLFIT (continued)

←ENTER ← USER DECIDES TO ENTER DATA FROM TTY
NØ. ØF POINTS TØ BE ENTERED ? 5
ENTER THE PØINTS IN X,Y ØRDER:
2.0,3.0 4.0,5.0 6.0,6.0 8.0,4.5 10.0,12.0

←STAT ← USER ASKS FOR STATISTICS ON HIS DATA

NUMBER ØF PØINTS= 5
MEAN VALUE ØF X= 6
MEAN VALUE ØF Y= 6.1
STD ERRØR ØF Y= 3.471310992

←PPØLFIT
INVALID CØMMAND
←PØLFIT ← USER ASKS TO HAVE A POLYNOMIAL "FIT" TO HIS DATA
TØ WHAT DEGREE ?
3 ← POLYNOMIAL WILL BE: $P_3(X) = A_1 + A_2X + A_3X^2 + A_4X^3$
PØLFIT ØF DEGREE 3 INDEX ØF DETERMINATIØN 0.967763544

←CØEF ← USER ~~ASKS~~ TO SEE COEFFICIENTS $(A_1, A_2, A_3, \text{AND } A_4)$
A(1) = -8.900000028
A(2) = 8.886904781
A(3) = -1.723214289
A(4) = 0.104166667

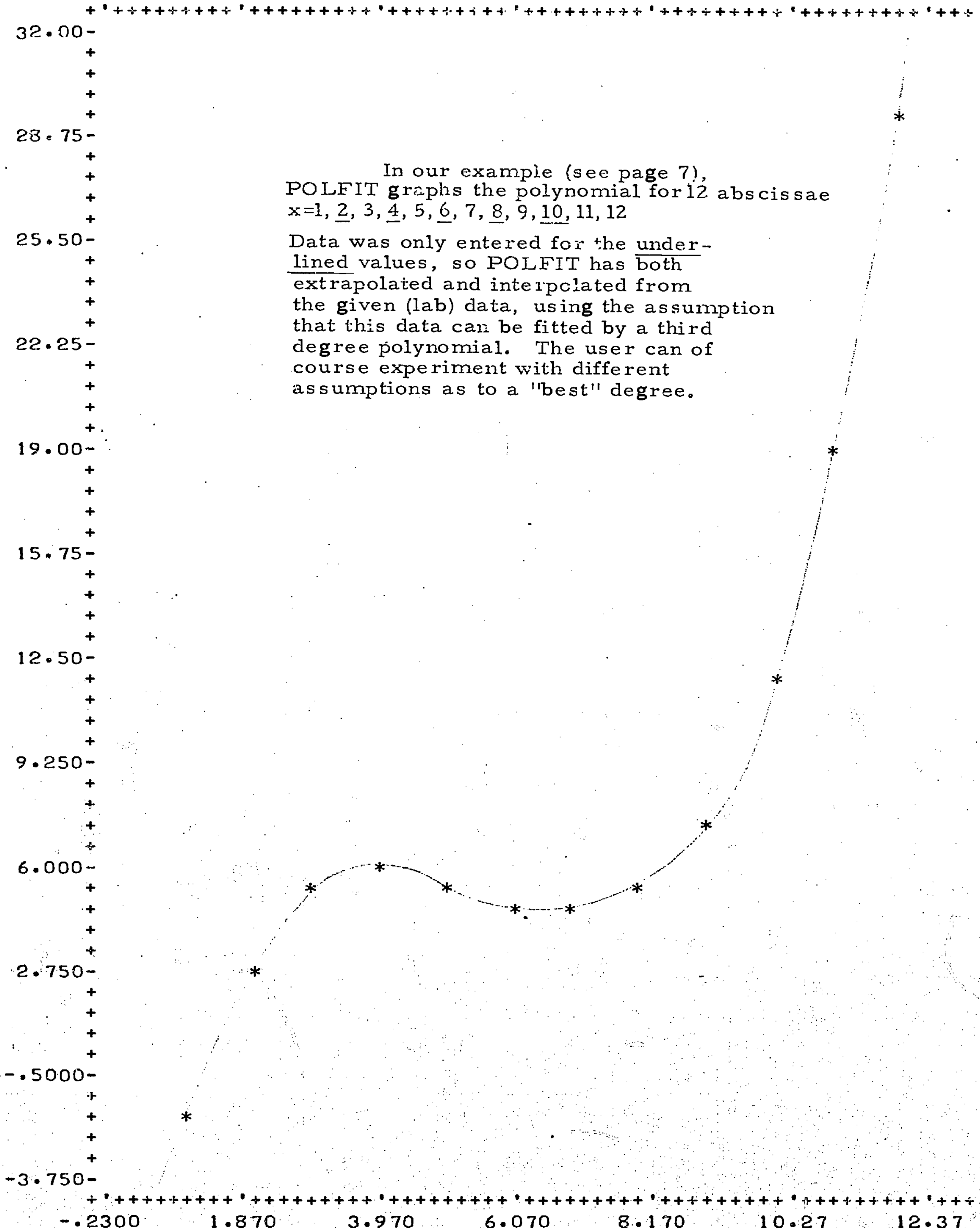
$P_3(x) = -8.9 + 8.88X - 1.72X^2 + 0.104X^3$

←SUMMARY ← USER ASKS TO SEE HOW WELL A 3RD DEGREE POLYNOMIAL FITS HIS 5 DATA POINTS
A(1) = -8.900000028
A(2) = 8.886904781
A(3) = -1.723214289
A(4) = 0.104166667

X-ACT	Y-ACT	Y-CALC	DIFF	PCT
.20000000E+01	.30000000E+01	.28142857E+01	.18571429E+00	.61904763E+01
.40000000E+01	.50000000E+01	.57428571E+01	-.7428571E+00	-.1485714E+02
.60000000E+01	.60000000E+01	.48857143E+01	.11142857E+01	.18571429E+02
.80000000E+01	.45000000E+01	.52428571E+01	-.7428571E+00	-.1650793E+02
.10000000E+02	.12000000E+02	.11814286E+02	.18571428E+00	.15476190E+01

STD ERRØR ØF ESTIMATE FØR Y= 0.881418514

←CHECK ← SEE EXPLANATION ABOVE — (THIS ALLOWS PLOTTING ANY NUMBER OF POINTS GENERATED BY YOUR POLYNOMIAL OVER ANY RANGE OF X)
MINIMUM VALUE ØF X=1
MAXIMUM VALUE ØF X=12
INCREMENT ØF X=1
PLØT ØR LIST? PLØT



← EXIT

← END

POLFIT (concluded)

-CØPY D01 TØ TEL

THE FØLLØWING IS A LIST ØF THE VARIØUS DEMØNSTRATIØN PRØGRAMS THAT ARE CURRENTLY AVAILABLE ØN THE CØM-SHARE SYSTEM.

FILE	NAME	SUBSYS	DESCRIPTIØN
D02	SNØDEMØ	SNØBØL	NUM (=100); LØAD D02, RUN
D03	HØRSES	BASIC	LØAD, RUN
D04	DICE	BASIC	LØAD, RUN
D05	BLACKJACK	BASIC	LØAD, RUN
D06	GØLF	BASIC	LØAD, RUN
D07	ØNE-ARM	CAL	LØAD, RUN
D08	DATE	FØS	RUN
D09	BASEBALL	FØS	RUN
D10	GØ-MØKU	EXE	EXE D10
D11	TAX	CAL	LØAD RUN
D12	CHARPUNCH	EXE	EXE D12
LIZ	SNØDEMØ	EXE	EXE LIZ

NOTE TO TUTORIAL AUTHORS:

For the ultimate in "reinforcement" try:

-NBS
>RUN 166TD /WHOOP/

and

-NBS
>RUN 166TD /CHIMES/

Math teachers should also look at /CHIMES/ to see an example of the use of modular arithmetic that your students can easily understand. It should be very easy to give a lecture to students on the subjects of "congruence" and "residue classes" after they have run this program.

Notes:
LIZ is the famous ELIZA program developed at MIT that tries to give the impression that there is a psychiatrist at the other end of the line. WARNING 1. Use of LIZ may be hazardous to your (emotional) health, and 2. the only way to terminate LIZ is by pressing Control G.
CHARPUNCH produces tapes like the HAPPY NEW YEAR enclosed.

The other programs do about what you might guess.

EXAMPLES OF USE:

1) -EXE LIZ
(blah blah blah)
G^c
****

2) -EXE D12
ENTER YOUR LINE.
HAPPY NEW YEAR␣ <—(This last space is necessary)
TURN PUNCH ON. (Push ON button on paper tape punch)
x ?? ?x
)FQ (These strange characters are printed while your tape is being punched. Turn the punch off about one second after you see the FQ)
ESC
****

3) -BASIC
>LOAD D04 (Brings in dice game - no // symbols used on system files)
>RUN (Causes dice game to execute)
....
ESC or EXIT or G^c

Project Solo Office: 682-6642, 621-3349. Peabody Office: 661-2900,TTY:
Allderdice Office: 521-2100, TTY's:422-7223, 422-7329, 422-7526,DPT:422-7416
Westinghouse Office: 661-7033, TTY: 363-2544.

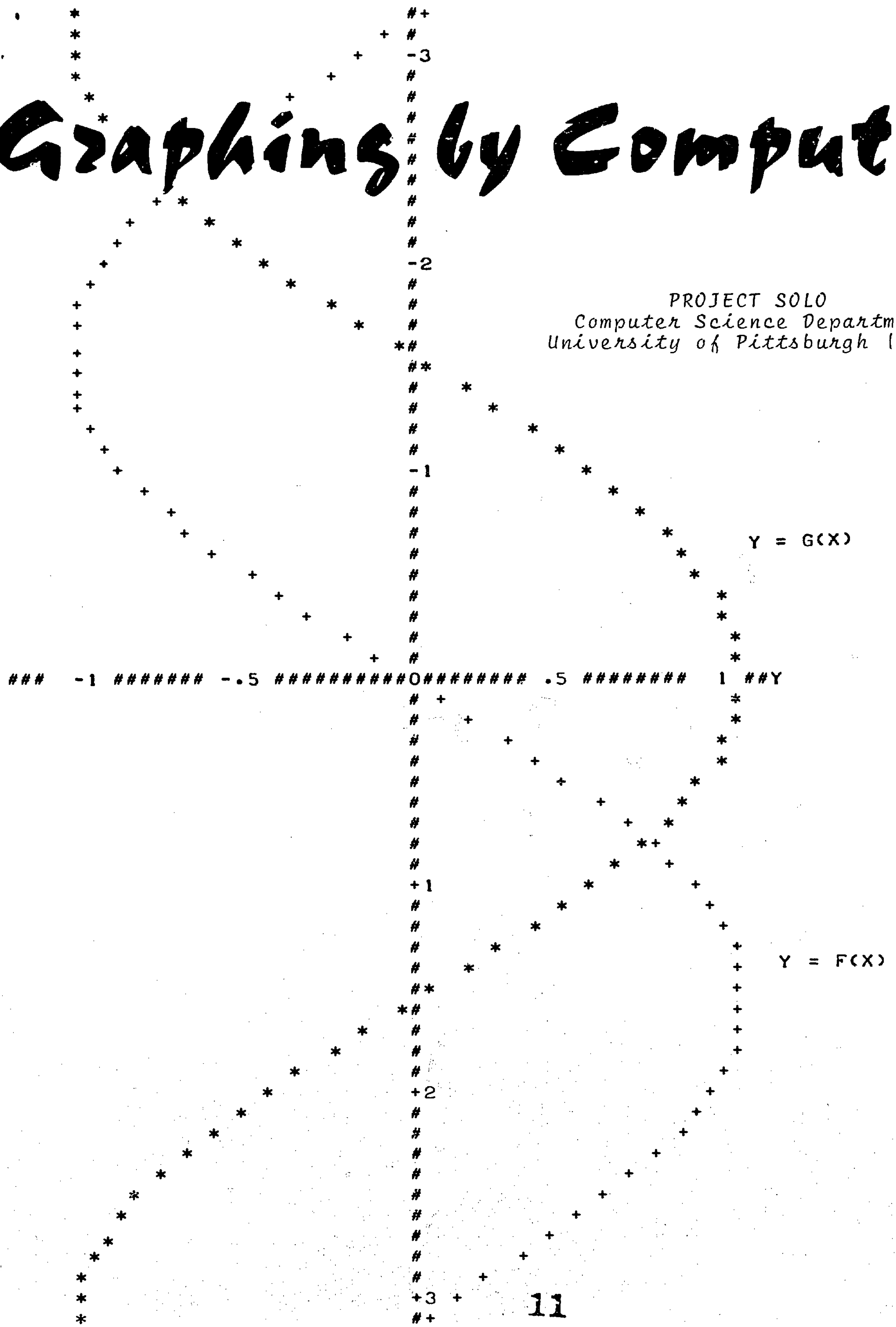
STAFF/SCHOOL VISITING SCHEDULE

(Name-School) A = Allderdice, W = Westinghouse, P = Peabody

	MONDAY - 1:30	TUESDAY - 1:30	WEDNESDAY - 1:30	THURSDAY - 1:30
OCT	19	20 Frank Wimberly-A Carolyn Len-A	21 Dr. Dwyer-W Dr. Bell-W	22
	26 Dr. Dwyer-Fox Chapel Dr. Bell-Fox Chapel	27 David Sironi-A Carolyn Len-A	28 Dr. Dwyer-A	29 Joanna Phillips-W Emilie Zielinski-W
NOV	2	3 David Sironi-A Frank Wimberly-A	4 Dr. Dwyer-W	5 Joanna Phillips- Emilie Zielinski-
	9	10 Frank Wimberly-A Carolyn Len-A	11 HOLIDAY	12 -Fox Chapel
	16	17 David Sironi-A(P?) Carolyn Len-A(P?)	18 Dr. Dwyer-P(?)	19 Joanna Phillips- E. Zielinski-W
	23 -Fox Chapel	24 David Sironi-W Carolyn Len-W	25 Dr. Dwyer-P	26 Frank Wimberly-A E. Zielinski-A
DEC	30	1 Frank Wimberly-W (P?) David Sironi-W(P?)	2 Dr. Dwyer-A	3 Joanna Phillips- Carolyn Len-A
	7 -Fox Chapel	8 David Sironi-W Frank Wimberly-W	9	10 Joanna Phillips- E. Zielinski-A
	14	15 David Sironi-W E. Zielinski-W	16	17 Joanna Phillips- E. Zielinski-A

Graphing by Computer

PROJECT SOLO
Computer Science Department
University of Pittsburgh (15213)



GRAPHING MATHEMATICAL FUNCTIONS

Graphs provide a useful means for investigating the properties of functions. Roots and maximum and minimum points can be estimated, regions in which the function is increasing or decreasing can be readily determined, and so on. Since calculating enough points to make a meaningful graph can be tedious and time-consuming, it is logical to consider applying the speed and accuracy of a computer to the problem. The purpose of this module is to illustrate the advantages and limitations of using a computer terminal in such applications.

After reviewing the graphing process using a second degree polynomial as an example, we will formulate a graphing algorithm. We will then write a computer program in NBS which uses this algorithm. Finally, we will suggest other graphing problems which will require that you write more sophisticated NBS graphing programs.

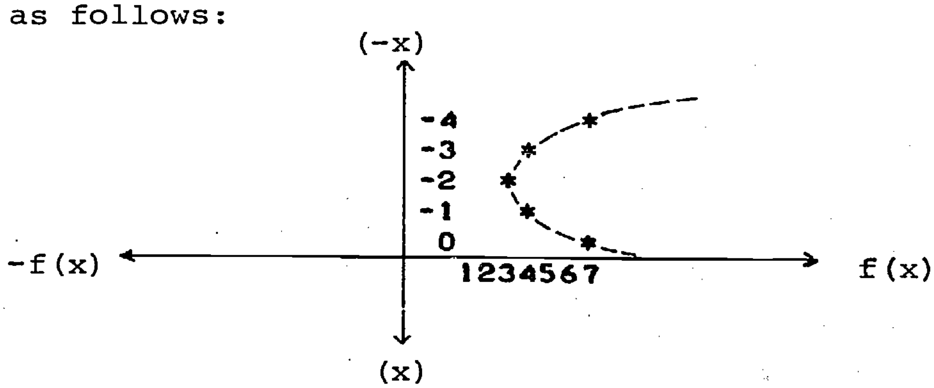
Graphing a Second Degree Polynomial

In order to graph a function f(x) we must obtain a list of ordered pairs of numbers of the form (a, f(a)); these correspond to points in the plane where a is the abcissa and f(a) is the ordinate. Recall that the set of possible values for a is called the domain and the set of possible values for f(a) is called the range.

Let's look at the function $f(x) = x^2 + 4x + 7$ on the domain $\{-4 \le x \le 0\}$. We must now find ordered pairs of numbers by substituting values of the domain into the function and finding the corresponding values of the range. We will do this only for the integers in the interval.

x	f(x)
-4	7
-3	4
-2	3
-1	4
0	7

Now we plot*these points on a rectangular coordinate graph as follows:



If we draw a smooth curve through the points we see that the graph of $f(x) = x^2 + 4x + 7$ is a parabola. Note that the domain $\{-4 \leq x \leq 0\}$ was well chosen since it contains the vertex (minimum point). What can you say about the roots of the equation $f(x) = x^2 + 4x + 7$?

<u>Formulation of the Algorithm</u>

At this point we will analyze what we have done in making a graph by hand into a number of steps without referring to a specific function such as $f(x) = x^2 + 4x + 7$.

The result will be an <u>algorithm</u> which can be applied in general.

---

*Rotate this sheet of paper 90° counter-clockwise to see the graph as shown in most textbooks. We have deliberately shown it in the form that would be printed on a computer.

Step 1. Choose a domain of interest. This will be an interval of the form $\{a \leq x \leq b\}$.

Step 2. Choose a step size d.

Step 3. Assign the value of a to the variable x.

Step 4. Calculate f(x).

Step 5. Place a mark at the point (x, f(x)) on the graph.

Step 6. Increase x by the step size d.

Step 7. If $x \leq b$ go to step 4; otherwise stop.

Note that steps 4 through 7 are repeated; hence f(x) is computed and the corresponding points plotted for x = a, a + d,... until x > b.

The NBS Program

Below is a computer program written in NBS, which uses the algorithm we developed, to produce the graph on page 2. Try the program by copying the statements exactly as shown. Try to relate the statements of the program to the steps of the algorithm.

```
10 LET A=-4
15 LET B=0
20 LET I=1
30 LET X=A
40 LET F=X↑2+4*X+7
50 PRINT TAB(10): X: TAB(F+11): "*"
60 LET X=X+I
70 IF X<=B GOTO 40
75 PRINT "            1234567"
80 END
```

"TAB(10)" in statement 50 causes X to be printed in the eleventh space; similarly, TAB(F+11) causes the "*" to be printed in the (F+12)th space (not the (10+F+12)th). In statement 75 be sure to leave 12 blanks before the 1. (Can you think of a better way to do this?)

EXPLORATION

The following exercises require that you write NBS programs which produce better graphs than the sample program can handle.

1.

```
########0#########1#########2#########3#########4#Y
        #                           *
        #                         *
        #                       *
        #                      *
        #                     *
        #                    *
        #                   *
        #                   *
        #                   *
        1                   *
        #                   *
        #                   *
        #                   *
        #                    *
        #                     *
        #                      *
        #                       *
        #                         *
        #                           *
        2                             *
```

Above is the graph of a parabola produced by an NBS program. Revise the program in the previous section so that it produces a comparable graph (in terms of number of points plotted, size, etc.) of the function $f(x) = x^2 + 4x + 7$ for $-4 \leq x \leq 0$.

2. Write a program to graph the function $f(x) = 0.1\ x^2 - 0.2x$ for $-1 \leq x \leq 3$.

<u>HINT</u>: Multiply the function by a "Scale Factor" and relabel the y-axis. How would you handle the problem of negative values of f(x)? Notice that the roots of the polynomial equation are values of x where the graph crosses the x-axis.

This means that f(x) = 0 at these points, which are therefore sometimes called "Zeroes" of the function.

3. Write a program to use in estimating the minimum value of the function $f(x) = (x^2 + 1)/x$, $0.2 \leq x \leq 4.0$. (Why wasn't x=0 included in the domain of f?)

4. Write a program to solve the pair of equations:

$$y = 0.9^x$$
$$y = 1/3\, x^2 - 1/2x$$

HINT 1: Write a program which graphs both functions at once; the solutions are the values of x where the graphs intersect.

HINT 2: In NBS, 0.9^x is written as: 0.9 ↑X
or as: 0.9 **X

GRAPHING BY COMPUTER - SAMPLE SOLUTIONS

These programs all use a For Loop instead of the "Increment--Go To" Loop used in the sample program in the module. The first part of each of the programs determines the character to be used for the X-axis A$--it is always either an integer or "#". The function is then computed and, when appropriate multiplied by a scale factor and added to a constant which is related to the distance of the X-axis from the left side of the paper. In the print statements,

TAB(F): "*"

causes * to be printed in the F + 1st space of the line regardless of other tabs in the print statement.

```
10 FØR I=-45 TØ 0
20 IF I<>0 THEN 30
23 PRINT "###0####1####2####3####4####5####6####7####Y"
72GØ TØ 170
30 IF I<>-40 THEN 33
31 LET A$="-4"
32 GØ TØ 120
33 IF I<>-30 THEN 36
34 LET A$="-3"
35 GØ TØ 120
36 IF I<>-20 THEN 39
37 LET A$="-2"
38 GØ TØ 120
39 IF I<>-10 THEN 100
40 LET A$="-1"
45 GØ TØ 120
100 LET A$="#"
120 LET X=I/10
130 LET F=X↑2+4*X+7
140 LET F=F*5
150 IF F<>0 THEN 155
151 PRINT TAB(3): "Ø"
152 GØ TØ 160
155 PRINT TAB(3): A$: TAB(3+F): "*"
160 NEXT I
170 END
```

(1. continued)

OUTPUT:

```
   #                                             *
   #                                          *
   #                                        *
   #                                      *
   #                                    *
   -4                                 *
   #                                *
   #                              *
   #                            *
   #                          *
   #                         *
   #                       *
   #                      *
   #                     *
   #                    *
   -3                  *
   #                  *
   #                 *
   #                *
   #               *
   #               *
   #              *
   #              *
   #              *
   #              *
   -2             *
   #              *
   #              *
   #              *
   #              *
   #               *
   #               *
   #                *
   #                 *
   #                  *
   -1                 *
   #                    *
   #                     *
   #                      *
   #                      *
   #                         *
   #                          *
   #                            *
   #                              *
   #                                *
###0####1####2####3####4####5####6####7####Y
```

2. Possible solution to problem 2.

PROGRAM:

```
10 FØR I=-10 TØ 30
20 IF I<>0 THEN 30
23 PRINT "###-.3##-.2##-.1####0###.1###.2###.3###.4####Y"
27 GØ TØ 160
30 IF I<>-10 THEN 33
31 LET A$="-1"
32 GØ TØ 120
33 IF I<>10 THEN 36
34 LET A$="+1"
35 GØ TØ 120
36 IF I<>20 THEN 39
37 LET A$="+2"
38 GØ TØ 120
39 IF I<>30 THEN 100
40 LET A$="+3"
45 GØ TØ 120
100 LET A$="#"
120 LET X=I/10
130 LET F=0.1*X↑2-0.2*X
140 LET F=F*50+20
150 IF F<>0 THEN 155
151 PRINT TAB(20): "Ø"
152 GØ TØ 160
155 IF F<20 THEN 158
156 PRINT TAB(20): A$: TAB(F): "*"
157 GØ TØ 160
158 PRINT TAB(F): "*": TAB(20): A$
160 NEXT I
170 END
```

```
                    -1             *
                    #            *
                    #          *
                    #        *
                    #      *
                    #     *
                    #   *
                    #  *
                    # *
                    #*
###-.3##-.2##-.1####0###.1###.2###.3###.4####Y
                   *#
                  * #
                 *  #
                *   #
                *   #
               *    #
               *    #
               *    #
               *    #
              *     +1
               *    #
               *    #
               *    #
               *    #
                *   #
                *   #
                 *  #
                  * #
                   *#
                    +2*
                    #*
                    # *
                    #  *
                    #   *
                    #     *
                    #      *
                    #        *
                    #          *
                    #            *
                    +3             *
```

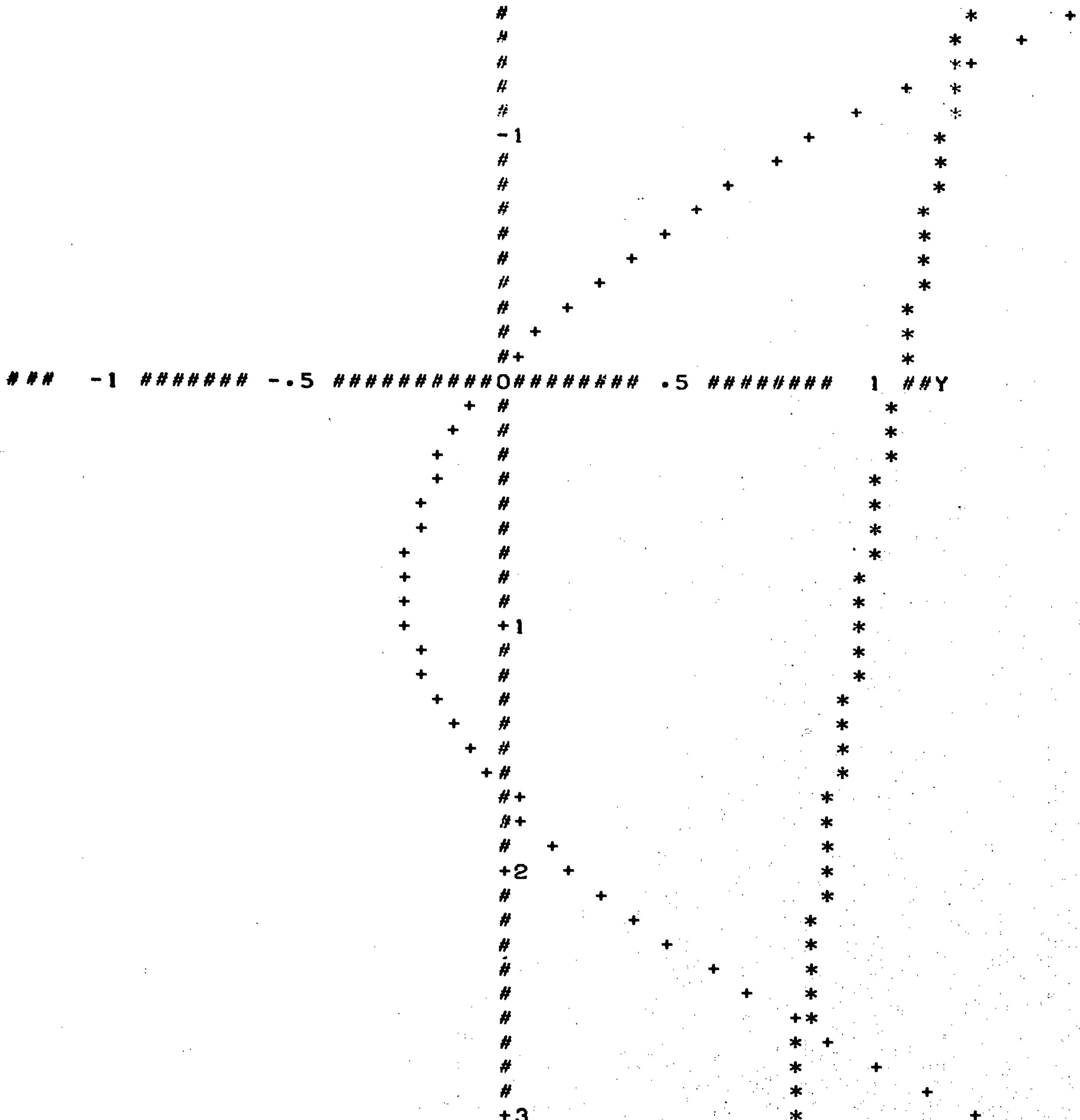
3. Sample solution to Problem 3.

```
10 PRINT "####0#########1#########2#########3#########4#####Y"
15 PRINT "    #"
20 FØR I=2 TØ 35
30 IF I<>10 THEN 50
40 LET A$="1"
45 GØ TØ 120
50 IF I<>20 THEN 70
60 LET A$="2"
65 GØ TØ 120
70 IF I<>30 THEN 100
80 LET A$="3"
90 GØ TØ 120
100 LET A$="#"
120 LET X=I/10
130 LET F=(X↑2+1)/X
140 LET F=F*10
150 PRINT TAB(4); A$; TAB(F+4); "*"
160 NEXT I
170 END
```

```
####0#########1#########2#########3#########4####Y
    #
    #                                                   *
    #                                   *
    #                            *
    #                        *
    #                     *
    #                    *
    #                   *
    #                   *
    1                   *
    #                   *
    #                   *
    #                   *
    #                    *
    #                    *
    #                     *
    #                     *
    #                      *
    #                       *
    2                        *
    #                        *
    #                         *
    #                          *
    #                           *
    #                           *
    #                            *
    #                             *
    #                              *
    #                               *
    3                                *
    #                                 *
    #                                  *
    #                                   *
    #                                   *
    #                                    *
```

4. Sample solution to Problem 4.

```
30 FØR I=-15 TØ 30
60 IF I<>-30 THEN 90
70 LET A$="-3"
80 GØ TØ 280
90 IF I<>-20 THEN 120
100 LET A$="-2"
110 GØ TØ 280
120 IF I<>-10 THEN 150
130 LET A$="-1"
140 GØ TØ 280
150 IF I<>10 THEN 180
160 LET A$="+1"
170 GØ TØ 280
180 IF I<>20 THEN 210
190 LET A$="+2"
200 GØ TØ 280
210 IF I<>30 THEN 270
220 LET A$="+3"
230 GØ TØ 280
270 LET A$="#"
280 IF I<>0 THEN 310
290 PRINT
"###  -1 ####### -.5 ##########0######## .5 ########
 1 ##Y"
300 GØ TØ 570
310 LET X=I/10
320 LET G=.9↑X
325 LET G=25*G+30
330 LET F=.3*X↑2-.5*X
332 LET F=F*25+30
335 IF F<G THEN 340
336 LET B$="*"
337 LET C$="+"
338 GØ TØ 350
340 LET B$="+"
345 LET C$="*"
350 LET H=MIN(F,G)
360 LET T=MAX(F,G)
370 IF F<>G THEN 470
380 LET B$="Ø"
390 IF F<>30 THEN 420
400 PRINT TAB(30): B$
410 GØ TØ 570
420 IF F<30 THEN 450
430 PRINT TAB(30): A$: TAB(F): B$
440 GØ TØ 570
450 PRINT TAB(F): B$: TAB(30): A$
460 GØ TØ 570
470 IF H>=30 THEN 560
480 IF T>=30 THEN 510
490 PRINT TAB(H): B$: TAB(T): C$: TAB(30): A$
500 GØ TØ 570
510 IF T=30 THEN 540
520 PRINT TAB(H): B$: TAB(30): A$: TAB(T): C$
530 GØ TØ 570
540 PRINT TAB(H): B$: TAB(30): A$
550 GØ TØ 570
560 IF H=30 THEN 563
561 PRINT TAB(30): A$: TAB(H): B$: TAB(T): C$
562 GØ TØ 570
563 PRINT TAB(30): B$: TAB(T): C$
570 NEXT I
580 END
```


-1
-1
-.5
O
.5
1
Y
1
2
3

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